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Emergent gauge fields and the high-temperature superconductors

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Citation	Sachdev, Subir. 2016. "Emergent Gauge Fields and the High-Temperature Superconductors." Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 374 (2075) (July 25): 20150248. doi:10.1098/rsta.2015.0248.
Published Version	doi:10.1098/rsta.2015.0248
Citable link	http://nrs.harvard.edu/urn-3:HUL.InstRepos:30778729
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arXiv:1512.00465v2 [cond-mat.str-el] 7 Dec 2015

PHILOSOPHICAL TRANSACTIONS A

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Research

Article submitted to journal

Subject Areas:
Condensed matter physics, superconductivity, magnetism

Keywords:
Gauge fields, Luttinger theorem, quantum entanglement

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Emergent gauge fields and the high temperature superconductors

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The quantum entanglement of many states of matter can be represented by electric and magnetic fields, much like those found in Maxwell's theory. These fields 'emerge' from the quantum structure of the many-electron state, rather than being fundamental degrees of freedom of the vacuum. I review basic aspects of the theory of emergent gauge fields in insulators in an intuitive manner. In metals, Fermi liquid theory relies on adiabatic continuity from the free electron state, and its central consequence is the existence of long-lived electron-like quasiparticles around a Fermi surface enclosing a volume determined by the total density of electrons, via the Luttinger theorem. However long-range entanglement and emergent gauge fields can also be present in metals. I focus on the 'fractionalized Fermi liquid' (FL*) state, which also has long-lived electron-like quasiparticles around a Fermi surface; however the Luttinger theorem on the Fermi volume is violated, and this requires the presence of emergent gauge fields, and the associated loss of adiabatic continuity to the free electron state. Finally, I present a brief survey of some recent experiments in the hole-doped cuprate superconductors, and interpret the properties of the pseudogap regime in the framework of the FL* theory.

Talk presented at *Unifying physics and technology in light of Maxwell's equations*, Discussion Meeting at the Royal Society, London, November 16-17, 2015, celebrating the 150th anniversary of Maxwell's equations.

arXiv:1512.00465.

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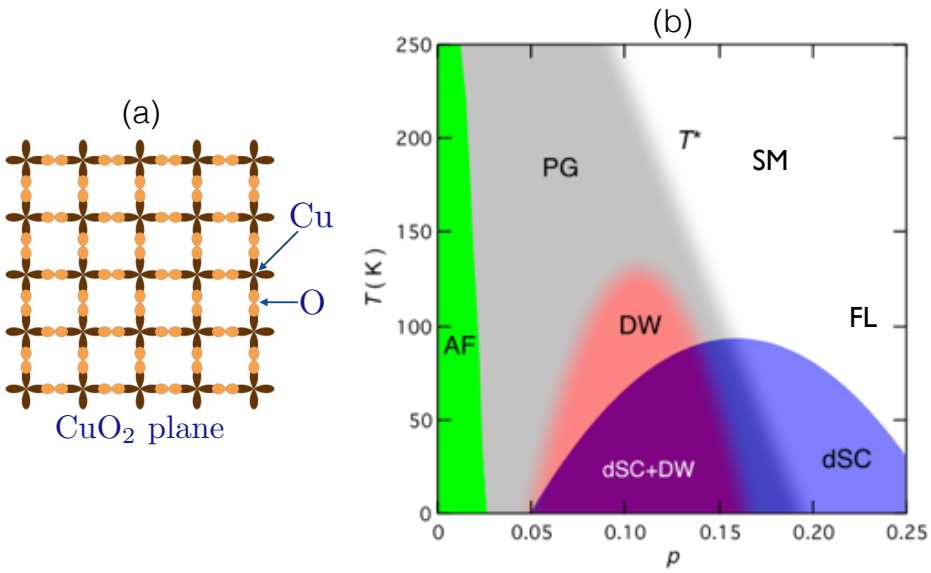

Figure 1. (a) The square lattice of Cu and O atoms found in every copper-based high temperature superconductor. (b) A schematic phase diagram of the YBCO superconductors as a function of the hole density p and the temperature T adapted from Ref. [6]. The phases are discussed in the text: AF–insulating antiferromagnet, PG–pseudogap, DW–density wave, dSC–d-wave superconductor, SM–strange metal, FL–Fermi liquid. The critical temperature for superconductivity is T_c, and T^* is the boundary of the pseudogap regime.

1. Introduction

The copper-based high temperature superconductors have provided a fascinating and fruitful environment for the study of quantum correlations in many-electron systems for over two decades. Significant experimental and theoretical advances have appeared at a steady pace over the years. In this article, I will review some theoretical background, and use it to interpret some remarkable recent experiments [1–8]. In particular, I argue that modern theoretical ideas on long-range quantum entanglement and emergent gauge fields provide a valuable framework for understanding the experimental results. I will discuss experimental signatures of quantum phases with emergent gauge fields, and their connections to the recent observations.

The common feature of all the copper-based superconductors is the presence of a square lattice of Cu and O atoms shown in Fig. 1a. For the purposes of this article, we can regard the O p orbitals as filled with pairs of electrons and inert. Only one of the Cu orbitals is active, and in a parent insulating compound, this orbital has a density of exactly one electron per site. The rest of this article will consider the physical properties of this Cu orbital residing on the vertices of a square lattice. It is customary to measure the density of electrons relative to the parent insulator with one electron per site: we will use p to denote the hole density: *i.e.* such a state has a density of $1 - p$ electrons per Cu site. A recent schematic phase diagram of the hole-doped superconductor YBCO is shown in Fig. 1b as a function of p and the temperature T. The initial interest in these compounds was sparked by the presence of high temperature superconductivity, indicated by the large values of T_c in Fig. 1b. However, I will not discuss the origin of this superconductivity in this article. Rather, the focus will be on the other phases, and in particular the pseudogap metal (PG in Fig. 1b): the physical properties of this metal differ qualitatively from those of conventional metals, and so are of significant intrinsic theoretical interest. Furthermore, superconductivity

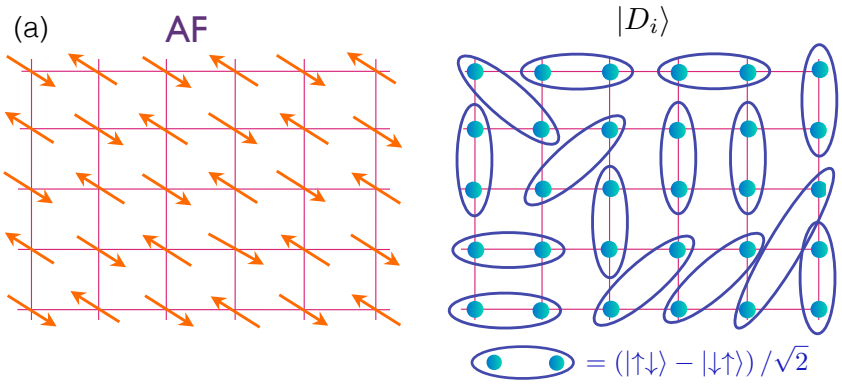

Figure 2. (a) The insulating AF state at $p = 0$. (b) Component of a 'resonating valence bond' wavefunction for the antiferromagnet which preserves spin rotation symmetry; all the $|D_i\rangle$ in Eq. (2.1) have similar pairings of electrons on nearby sites (not necessarily nearest-neighbor).

appears as a low temperature instability of the pseudogap, so a theory of the high value of T_c can only appear after a theory of the PG metal.

We begin our discussion by describing the simpler phases at the extremes of p in Fig. 1b.

At (and near) $p = 0$, we have the antiferromagnet (AF) which is sketched in Fig. 2a. The Coulomb repulsion between the electrons keeps their charges immobile on the Cu lattice sites, so that each site has exactly one electron. The Coulomb interaction is insensitive to the spin of the electron, and so it would appear that each electron spin is free to rotate independently on each site. However, there are virtual 'superexchange' processes which induce terms in the effective Hamiltonian which prefer opposite orientations of nearest-neighbor spins, and the optimal state turns out to be the antiferromagnet (AF) sketched in Fig. 2a. In this state, the spins are arranged in a checkerboard pattern, so that all the spins in one sublattice are parallel to each other, and anti-parallel to spins on the other sublattice. Two key features of this AF state deserve attention here: (*i*) The state breaks a global spin rotation symmetry, and essentially all of its low energy properties can be described by well-known quantum field theory methods associated with spontaneously broken symmetries. (*ii*) The wavefunction does not have long-range entanglement, and the exact many-electron wavefunction can be obtained by a series of local unitary transformations on the simple product state sketched in Fig. 2a.

At the other end of larger values of p, we have the Fermi liquid (FL) phase. This is a metallic state, in which the electronic properties are most similar to those of simple monoatomic metals like sodium or gold. This is also a quantum state without long-range entanglement, and the many-electron wavefunction can be well-approximated by a product over single electron momentum eigenstates (Bloch waves); note the contrast from the AF state, where the relevant single-particle states were localized on single sites in position space. We will discuss some further important properties of the FL state in Section 3.

Section 2 will describe possible insulating states on the square lattice, other than the simple AF state found in the cuprate compounds at $p = 0$. The objective here will be to introduce states with long-range quantum entanglement in a simple setting, and highlight their connection to emergent gauge fields. Then Section 4 will combine the descriptions of Sections 3 and 2 to propose a metallic state with long-range quantum entanglement and emergent gauge fields: the fractionalized Fermi liquid (FL*). Finally, in Section 5, we will review the evidence from recent experiments that the pseudogap (PG) regime of Fig. 1b is described by a FL* phase.

I also note here another recent review article [9], which discusses similar issues at a more specialized level aimed at condensed matter physicists. The gauge theories of the insulators discussed in Section 2 were reviewed in earlier lectures [10, 11].

2. Emergent gauge fields in insulators

The spontaneously broken spin rotation symmetry of the AF state at $p = 0$ is not observed at higher p. This section will therefore describe quantum states which preserve spin rotation symmetry. However, in the interests of theoretical simplicity, we will discuss such states in the insulator at the density of $p = 0$, and assume that the AF state can be destabilized by suitable further-neighbor superexchange interactions between the electron spins.

We begin with the 'resonating valence bond' (RVB) state

$$|\Psi\rangle = \sum_i c_i |D_i\rangle \tag{2.1}$$

where i extends over all possible pairings of electrons on nearby sites, and a state $|D_i\rangle$ associated with one such pairing is shown in Fig. 2b; the c_i are complex co-efficients we will leave unspecified here. Note that the electrons in a valence bond need not be nearest-neighbors. Each $|D_i\rangle$ is a spin singlet, and so spin rotation invariance is preserved; the antiferromagnetic exchange interaction is optimized between the electrons within a single valence bond, but not between electrons in separate valence bonds. We also assume that the c_i respect the translational and other symmetries of the square lattice. Such a state was first proposed by Pauling [12] as a description of a simple metal like lithium. We now know that Pauling's proposal is incorrect for such metals. But we will return to a variant of the RVB state in Section 4 which does indeed describe a metal, and this metal will be connected to the phase diagram of the cuprates in Section 5. Anderson revived the RVB state many years later [13] as a description of Mott insulators: these are materials with a density of one electron per site, which are driven to be insulators by the Coulomb repulsion between the electrons (contrary to the Bloch theorem for free electrons, which requires metallic behavior at this density).

In a modern theoretical framework, we now realize that the true significance of the Pauling-Anderson RVB proposal was that it was the first ansatz to realize *long-range* quantum entanglement. Similar entanglement appeared subsequently in Laughlin's wavefunction for the fractional quantum Hall state [14], and for RVB states in the absence of time-reversal symmetry [15]. The long-range nature of the entanglement can be made precise by computation of the 'topological entanglement entropy' [16–18]. But here we will be satisfied by a qualitative description of the sensitivity of the spectrum of states to the topology of the manifold on which the square lattice resides. The sensitivity is present irrespective of the size of the manifold (provided it is much larger than the lattice spacing), and so indicates that the information on the quantum entanglement between the electrons is truly long-ranged. A wavefunction which is a product of localized single-particle states would not care about the global topology of the manifold.

The basic argument on the long-range quantum information contained in the RVB state is summarized in Fig. 3. Place the square lattice on a very large torus (*i.e.* impose periodic boundary conditions in both directions), draw an arbitrary imaginary cut across the lattice, indicated by the red line, and count the number of valence bonds crossing the cut. It is not difficult to see that any *local* re-arrangement of the valence bonds will preserve the number of valence bonds crossing the cut modulo 2. Only very non-local processes can change the parity of the valence bonds crossing the cut: one such process involves breaking a valence bond across the cut into its constituent electrons, and moving the electrons separately around a cycle of the torus crossing the cut, so that they meet on the other side and form a new valence bond which no longer crosses the cut–see Fig. 4. Ignoring this very non-local process, we see that the Hilbert space splits into disjoint sectors, containing states with even or odd number of valence bonds across the cut [19, 20]. Locally, the two sectors are identical, and so we expect the two sectors to have ground states (and also excited states) of nearly the same energy for a large enough torus. The presence of these near-degenerate states is dependent on the global spatial topology, *i.e.* it requires periodic boundary conditions around the cycles of the torus, and so can be viewed as a signature of long-range quantum entanglement.

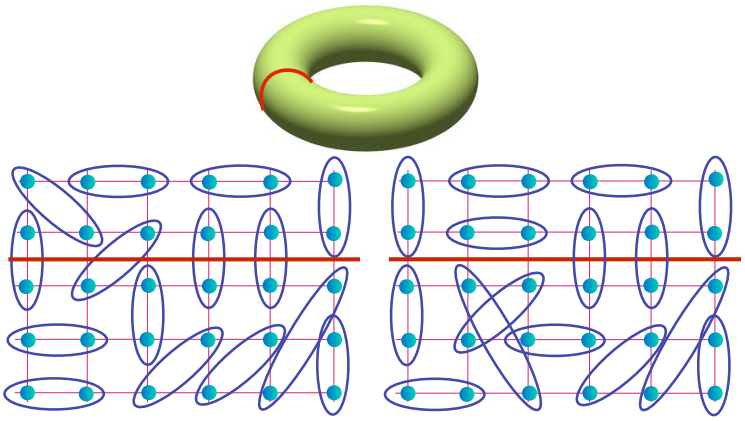

Figure 3. Sensitivity of the RVB state to the torus geometry: the number of valence bonds crossing the cut (red line) can only differ by an even integer between any two configurations (like those shown) which differ by an arbitrary local arrangement of valence bonds.

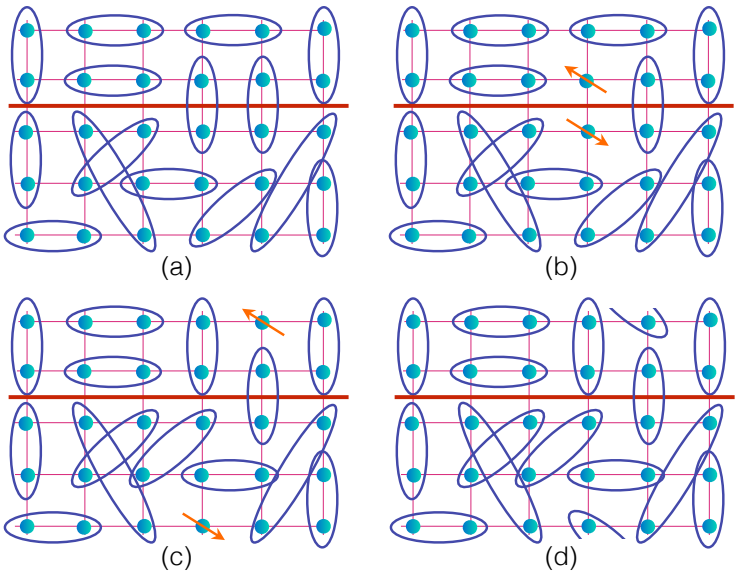

Figure 4. Non-local process which changes the parity of the number of valence bonds crossing the cut. A valence bond splits into two spins, which pair up again after going around the torus.

The above description of topological degeneracy and entanglement relies on a somewhat arbitrary and imprecise trial wavefunction. A precise understanding is provided by a formulation of the physics of RVB in terms of an emergent gauge theory, the first example of which was introduced by Baskaran and Anderson [21]. Such a formulation provides another way to view the nearly-degenerate states obtained above on a torus: they are linear combinations of states obtained by inserting fluxes of the emergent gauge fields through the cycles of the torus.

The formulation as a gauge theory [21, 22] becomes evident upon considering a simplified model with valence bonds only between nearest-neighbor sites on the square lattice. We introduce valence bond number operators $\hat{n}$ on every nearest-neighbor link, and then there is a crucial

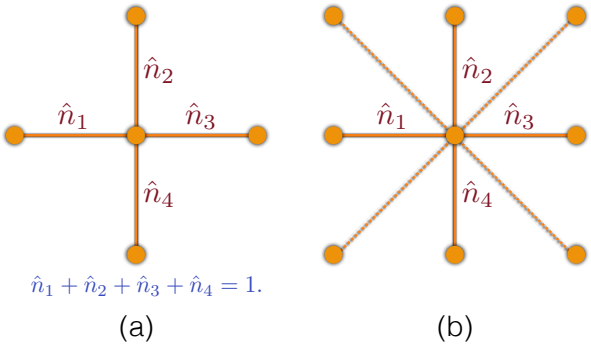

Figure 5. (a) Nearest-neighbor valence bond number operators, proportional to the electric field of a compact U(1) gauge theory. (b) Model with valence bonds connecting the same sublattice: now the constraint on the number operators is modified, and the spin liquid is described by a $\mathbb{Z}_2$ gauge theory.

constraint that there is exactly one valence bond emerging from every site, as illustrated in Fig. 5a. After introducing oriented 'electric field' operators $\hat{E}_{i\alpha} = (-1)^{i_x+i_y}\hat{n}_{i\alpha}$ (here i labels sites of the square lattice, and $\alpha = x, y$ labels the two directions), this local constraint can be written in the very suggestive form [22]

$$\Delta_\alpha \hat{E}_{i\alpha} = \rho_i, \tag{2.2}$$

where Δ_α is a discrete lattice derivative, and $\rho_i \equiv (-1)^{i_x+i_y}$ is a background 'charge' density. Eq. (2.2) is analogous to Gauss's Law in electrodynamics, and a key indication that the physics of resonating valence bonds is described by an emergent gauge theory. An important difference from Maxwell's U(1) electrodynamics is that the eigenvalues of the electric field operator $\hat{E}_{i\alpha}$ must be integers. In terms of the canonically conjugate gauge field $\hat{A}_{i\alpha}$

$$[\hat{A}_{i\alpha}, \hat{E}_{j\beta}] = i\hbar\delta_{ij}\delta_{\alpha\beta}, \tag{2.3}$$

the integral constraint translates into the requirement that $\hat{A}_{i\alpha}$ is a compact angular variable on a unit circle, and that $\hat{A}_{i\alpha}$ and $\hat{A}_{i\alpha} + 2\pi$ are equivalent. So there is an equivalence between the quantum theory of nearest-neighbor resonating valence bonds on a square lattice, and compact U(1) electrodynamics in the presence of fixed background charges ρ_i [22]. A non-perturbative analysis of such a theory shows [23, 24] that ultimately there is no gapless 'photon' associated with the emergent gauge field $\hat{A}$: compact U(1) electrodynamics is confining in 2 spatial dimensions, and in the presence of the background charges the confinement leads to valence bond solid (VBS) order illustrated in Fig. 6. The VBS state breaks square lattice rotation symmetry, and all excitations of the antiferromagnet, including the incipient photon, have an energy gap. In subsequent work, it was realized that the gapless photon can re-emerge at special 'deconfined' critical points [26–28] or phases [29], even in 2 spatial dimensions. In particular, in certain models with a quantum phase transition between a VBS state and the ordered antiferromagnet in Fig. 2a [23, 24, 26], the quantum critical point supports a gapless photon (along with gapless matter fields). This is illustrated in Fig. 6b by numerical results of Sandvik [25]: the circular distribution of valence bonds is evidence for an emergent continuous lattice rotation symmetry, and the associated Goldstone mode is the dual of the photon.

Although U(1) gauge theory does realize spin liquids with long-range entanglement and emergent photons, the gaplessness and 'criticality' of the spin liquids indicates the presence of long-range valence bonds, and the Pauling-Anderson trial wavefunctions are poor descriptions of such states. A stable, gapped quantum state with time-reversal symmetry, long-range entanglement and emergent gauge fields was first established in Refs. [30–34] using a model with short-range valence bonds which also connect sites on the same sublattice (Fig. 5b). It was shown

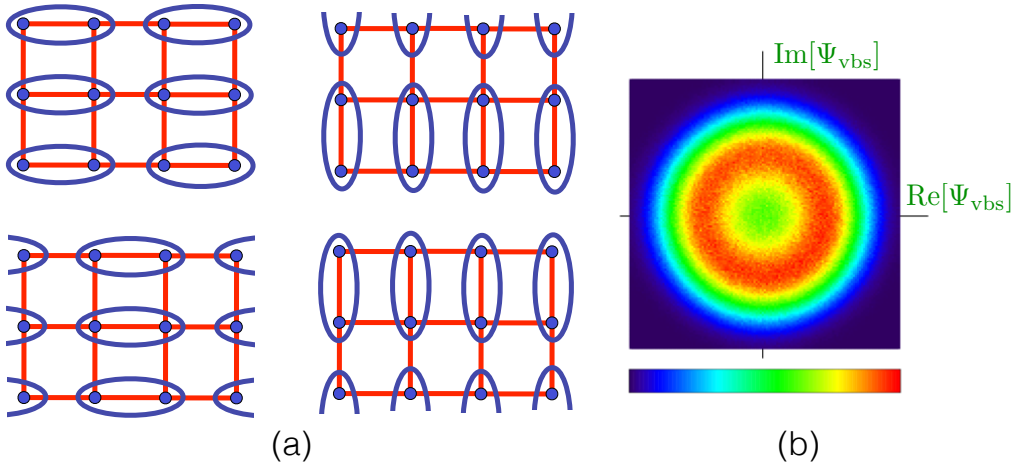

Figure 6. (a) The 4 VBS states which break square lattice rotational symmetry (b) Distribution of the complex VBS order parameter Ψ_{vbs} in the quantum Monte Carlo study by Sandvik [25]; the real and imaginary parts of this order measure the probability of the VBS states in the first and second columns. The near circular distribution of Ψ_{vbs} reflects an emergent symmetry which is a signature of the existence of a photon.

[30–34] that the same-sublattice bonds act like charge ± 2 Higgs fields in the compact U(1) gauge theory, and in such gauge theories there can be [35, 36] a 'Higgs' phase. Such a phase realizes a stable, gapped, RVB state preserving all symmetries of the Hamiltonian, including time-reversal, and is described by an emergent $\mathbb{Z}_2$ gauge theory [31, 34]. The $\mathbb{Z}_2$ gauge theory can be viewed as a discrete analog of the compact U(1) theory in which the gauge field takes only two possible values $\hat{A}_{i\alpha} = 0, \pi$. The intimate connection between a spin liquid with a deconfined $\mathbb{Z}_2$ gauge field, and a non-bipartite RVB trial wavefunction like Eq. (2.1), was shown convincingly by Wildeboer *et al.* [18]. Upon varying parameters in the underlying Hamiltonian, the $\mathbb{Z}_2$ spin liquid can undergo a confinement transition to a VBS phase which is described by a dual frustrated Ising model [31, 34]. Since these early works, the $\mathbb{Z}_2$ spin liquid has appeared in a number of other models [37–41], including the exactly solvable 'toric code' [39].

There is an interesting analogy between the theory of $\mathbb{Z}_2$ spin liquids and that of the $\mathbb{Z}_2$ topological insulator proposed more recently by Kane and Mele [42]. The $\mathbb{Z}_2$ topological insulator is the time-reversal invariant analog of the *integer* quantum Hall state of fermions at filling fraction $\nu = 1$, and the former is obtained, roughly speaking, by taking two copies of the latter. Kalmeyer and Laughlin [15] asserted that RVB states of spin systems must also break time-reversal symmetry and are analogous to the *fractional* quantum Hall state of bosons at filling fraction $\nu = 1/2$. However, it was subsequently shown [30–34] that a time-reversal invariant RVB state is possible, and this is the $\mathbb{Z}_2$ spin liquid discussed above. Interestingly, the $\mathbb{Z}_2$ spin liquid also has a connection to the theory of 2 copies of the $\nu = 1/2$ quantum Hall state. These connections between quantum Hall states, topological insulators, and spin liquids can be made precise using abelian Chern-Simons gauge theories [41, 43].

3. The Fermi liquid

We now turn to the metallic state found at large p (above the superconducting T_c) in Fig. 1b. This is the familiar Fermi liquid (FL), similar to that found in simple metals like sodium or gold.

The key properties of a Fermi liquid, reviewed in many text books are:

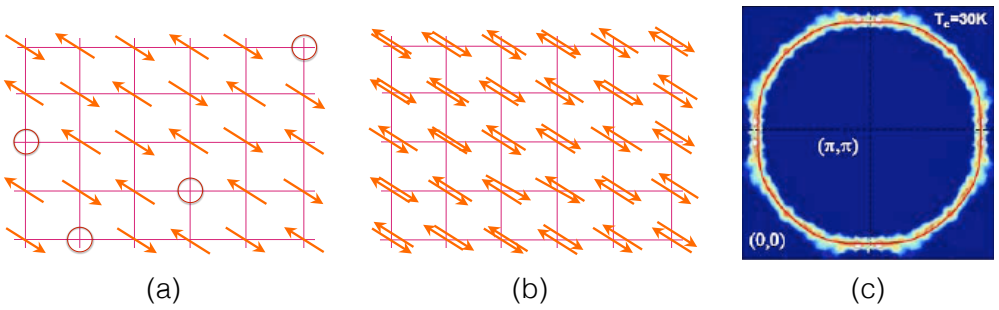

Figure 7. (a) State obtained after removing electrons with density p from the AF state in Fig. 2a. Relative to the fully-filled state with 2 electrons per site in (b), this state has a density of holes equal to $1+p$. (c) Photoemission results from Ref. [44] showing a Fermi surface of size $1+p$ in the FL region of Fig. 1b. This is the expected Luttinger volume at this density, in a state without any antiferromagnetic order.

- The Fermi liquid state of interacting electrons is adiabatically connected to the free electron state. The ground state of free electrons has a Fermi surface in momentum space, which separates the occupied and empty momentum eigenstates. This Fermi surface is also present in the interacting electron state, and the low energy excitations are long-lived, electron-like quasiparticles near the Fermi surface.
- The Luttinger theorem states that the volume enclosed by the Fermi surface (*i.e.* the Fermi volume) is equal (modulo phase space factors we ignore here) to the total density of electrons. This equality is obviously true for free electrons, but is also proven to be true, to all orders in the interactions, to any state adiabatically connected to the free electron state.
- For simple convex Fermi surfaces, the Fermi volume can be measured by the Hall coefficient, R_H, measuring the transverse voltage across a current in the presence of an applied magnetic field. We have $1/(eR_H) = -(\text{electron density})$ for electron-like Fermi surfaces, and $1/(eR_H) = (\text{hole density})$ for hole-like Fermi surfaces.

For the cuprates, the Fermi liquid is obtained by removing a density, p, of electrons from the insulating antiferromagnet, as shown in Fig. 7a. Relative to the fully-filled state with 2 electrons on each site (Fig. 7b), this state has a density of holes equal to $1+p$. Hence the Fermi liquid state without antiferromagnetic order can have a single hole-like Fermi surface with a Fermi volume of $1+p$ (and *not* p). And indeed, just such a Fermi surface is observed in photoemission experiments in the cuprates (Fig. 7c) in the region marked FL in Fig. 1b.

4. Emergent gauge fields in a metal: the fractionalized Fermi liquid

Next, we turn our attention to the smaller p region marked PG ("pseudogap") in Fig. 1b. We will review the experimental observations in this region in Section 5, but for now we note that in many respects this region behaves like an ordinary Fermi liquid, but with the crucial difference that the density of charge carriers is p and not the Luttinger density of $1+p$. So here we ask the theoretical question: is it possible to obtain a Fermi liquid which violates the Luttinger theorem and has a Fermi surface of size p of electron-like quasiparticles? From the structure of the Luttinger theorem we know that any such state cannot be adiabatically connected to the free electron state. A key result is that long-range quantum entanglement and associated emergent gauge fields are *necessary* characteristics of metallic states which violate the Luttinger theorem, and these also break the adiabatic connection to the free electron state [45, 46].

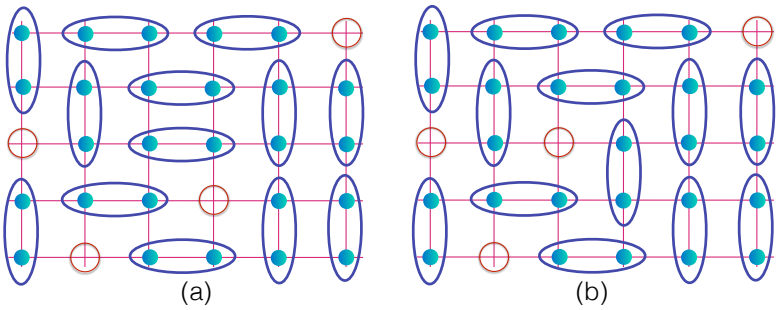

Figure 8. (a) State obtained by pairing the spins in Fig. 7a into valence bonds. (b) Resonance between the valence bonds leads to the motion of the vacancy in the center of the figure. The mobile vacancy is a 'holon', carrying unit charge but no spin. If the holons have fermionic statistics, such a mobile holon state can realize a holon metal. Only nearest-neighbor pairs of spins are shown for simplicity.

(There are claims [47] that zeros of electron Green's functions can be used to modify the Luttinger result. I believe such results are artifacts of simplified models. Such zeros do not generically exist as lines in the Brillouin zone (in two dimensions) for gapless states, because both the real and imaginary parts of the Green's functions have to vanish.)

One way to obtain a metal with carrier density p, and without antiferromagnetic order is to imagine that the electron spins in Fig. 7a pair up into resonating valence bonds, rather like the insulator in Fig. 2b. This is illustrated in Fig. 8a; the resonance between the valence bonds can now allow processes in which the vacant sites can move, as shown in Fig. 8b. As this process now transfers physical charge, the resulting state can be expected to be electrical conductor. A subtle computation is required to determine the quantum statistics obeyed by the mobile vacancies, but depending upon the parameter regimes, it can be either bosonic or fermionic [48, 49]. Assuming fermionic statistics, we have the possibility that the vacancies will form a Fermi surface, realizing a metallic state. Note that the vacancies do not transport spin, and such spinless charge carriers are often referred to as 'holons'; the metallic state we have postulated is a holon metal. The low energy quasiparticles near the Fermi surface of the holon metal will also be holons, carrying unit electrical charge but no spin. Consequently, such quasiparticles are not directly observable in photoemission experiments, which necessarily eject bare electrons with both charge and spin. As low energy electronic quasiparticles are observed in photoemission studies of the PG region in the cuprates (see Section 5), the holon metal is not favored as a candidate for the PG metal.

To obtain a spinful quasiparticle, we clearly have to attach an electronic spin to each holon. And as shown in Fig. 9, it is not difficult to imagine conditions under which this might be favorable: (*i*) We break density $p/2$ valence bonds into their constituent spins (Fig. 9a); this costs some exchange energy for each valence bond broken. (*ii*) We move the constituent spins ('spinons') into the neighborhood of the holons. (*iii*) The holons and spinons form a bound state (Fig. 9b) which has both charge $+e$ and spin $S = 1/2$, the same quantum numbers as (the absence of) an electron; this bound state formation gains energy which can offset the energy cost of (*i*). We now have a modified resonating valence bond state [50], like that in Eq. (2.1), but with $|D_i\rangle$ consisting of pairing of sites of the square lattice with two categories of 'valence bonds': the blue and green dimers in Fig. 9b. The first class (blue) are the same as the electron singlet pairs found in the Pauling-Anderson state. The second class (green) consists of a single electron resonating between the two sites at the ends of the bond. From their constituents, it is clear that relative to the insulating RVB state, the blue dimers are spinless, charge neutral bosons, while the green dimers are spin $S = 1/2$, charge $+e$ fermions. Evidence that the states associated with the blue and green dimers dominate the wavefunction of the lightly-doped cuprates appears in cluster dynamical mean field studies [51, 52]. Both classes of dimers are mobile, and situation

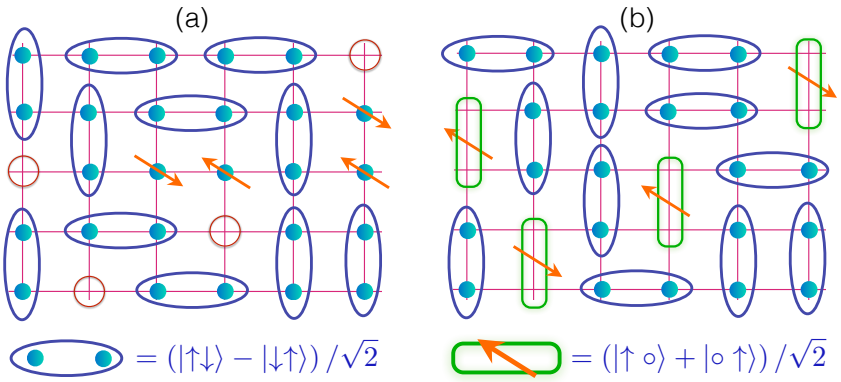

Figure 9. (a) State obtained by breaking density $p/2$ valence bonds in Fig. 8a into their constitute spinons. (b) The spinons move into the neighborhood of the vacancies and form holon-spinon bound states represented by the green dimers [50]. The state with resonating blue and green dimers realizes a metal with a Fermi volume of p quasiparticles with charge $+e$ and spin $S = 1/2$: the fractionalized Fermi liquid (FL*).

is somewhat analogous to ^{4}He-^{3}He mixture. Like the ^{3}He atoms, the green fermions can form a Fermi surface, and extension of the Luttinger argument to the present situation shows that the Fermi volume is exactly p [45, 53, 54]. However, unlike the ^{4}He-^{3}He mixture, superfluidity is not immediate, because of the close-packing constraint on the blue+green dimers; onset of superfluidity will require pairing of the green dimers, and will not be explored here. So the state obtained by resonating motion of the dimers in Fig. 9b is a metal, dubbed the fractionalized Fermi liquid (FL*) [53]. It has a Fermi volume of p, with well-defined electron-like quasiparticles near the Fermi surface.

A metallic state with a Fermi volume of density p holes with charge $+e$ and spin $S = 1/2$ was initially described in Refs. [55, 56] by considering a theory for the loss of antiferromagnetic order in a doped antiferromagnetic state like that in Fig. 7a. Numerous later studies [50, 54, 57–63] described the resulting metallic state more completely in terms of the binding of holons and spinons, similar to the discussion above. These studies also showed the presence of the emergent gauge excitations in the metal.

Here, we can see the presence of emergent gauge fields, and associated low energy states sensitive to the topology of the manifold, in our simplified description here of the FL* metal. Indeed, such low energy states are required to evade the Luttinger theorem on the Fermi surface volume [45, 46]. The FL* metal shares its topological features with corresponding insulating spin liquids, and we can transfer all of the arguments of Section 2 practically unchanged, merely by applying them to wavefunctions like Fig. 9b in a 'color blind' manner. So the arguments in Fig. 3 on the conservation of the number of valence bond across the cut modulo 2, and associated near-degeneracies on the torus, apply equally to the FL* wavefunction after counting the numbers of *both* blue and green dimers (see Fig. 10). The presence of these near-degenerate topological states is also crucial for the Luttinger-volume-violating Fermi surface. Oshikawa [64] presented a proof of the Luttinger volume in a Fermi liquid by considering the consequences of adiabatically inserting a fluxoid $\Phi = h/e$ through a cycle of the torus, while assuming that the only low energy excitations on the torus are the quasiparticles around the Fermi surface. However, with the availibility of the low energy topological states discussed in Fig. 10, which are not related to quasiparticle excitations, it is possible to modify Oshikawa's proof, and obtain a Fermi volume different from the Luttinger volume [45, 46]; indeed a Luttinger volume of p holes appears naturally in many models, including the simple models discussed here.

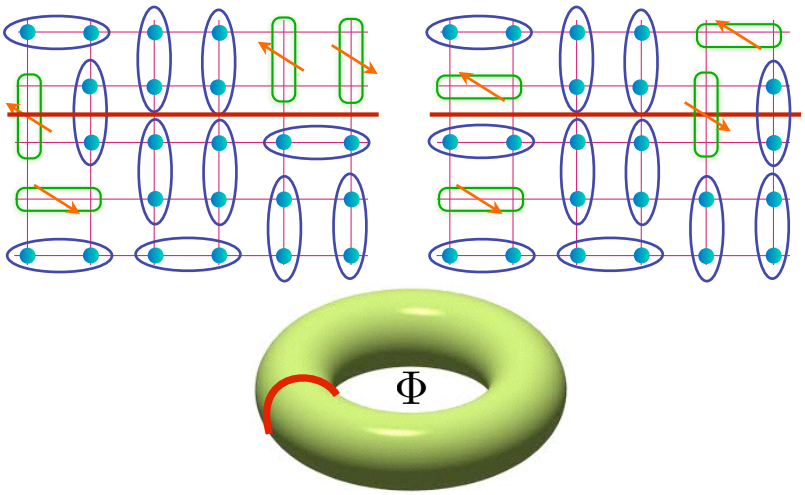

Figure 10. Sample configurations of the wavefunction of the FL* state. The two configurations differ by local rearrangments which preserve the sum of the number of blue *and* green dimers crossing the cut modulo 2, just as in Fig. 3 (only nearest-neighbor dimers are shown for simplicity). This conservation implies the presence of gauge excitations, and additional states sensitive to the global topology, which cannot be decomposed into the quasiparticle excitations around the Fermi surface. Arguments in Refs. [45, 46, 64], based upon the adiabatic insertion of a fluxoid $\Phi = h/e$ through a cycle of the torus, show that the presence of these states allow the Fermi volume to take a non-Luttinger value.

To summarize, this section has presented a simple description of a metallic state with the following features:

- A Fermi surface of holes of charge e and spin $S = 1/2$ enclosing volume p, and not the Luttinger volume of $1 + p$.
- Additional low energy quantum states on a torus not associated with quasiparticle excitations *i.e.* emergent gauge fields.

The flux-piercing arguments in Refs. [45, 46] show that it is not possible to have the first feature without the second.

5. The pseudogap metal of the cuprates

An early indication of the mysterious nature of the PG regime in Fig. 1b was its remarkable photoemission spectrum [65, 66] (Fig. 11). There are low energy electronic excitations along the 'nodal' directions in the form of 'Fermi arcs', but none in the anti-nodal directions. In the FL* proposal, these arcs are remnants of hole pockets centered on the Brillouin zone diagonals, with intensity suppressed on the 'back' sides of the pockets [47, 61], as shown in Fig. 11. The complete hole pockets have not been observed in photoemission, but this is possibly accounted for by thermal broadening and weak intensity on their 'back' sides.

More persuasive evidence for the FL* interpretation of the PG phase has come from a number of other experiments:

- A T-independent positive Hall co-efficient R_H corresponding to carrier density p in the higher temperature pseudogap [67]. This is the expected Hall co-efficient of the hole pockets in the FL* phase.

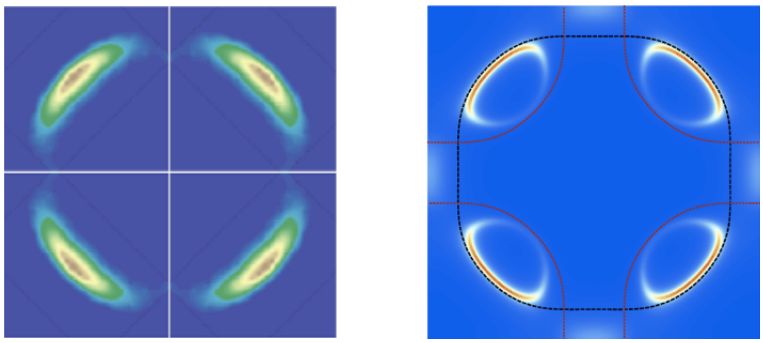

Figure 11. (a) Photoemission spectrum in the PG regime [65]. Shown is the first Brillouin zone of the square lattice centered at (π, π), as in Fig. 7c. However, unlike the FL result in Fig. 7c, there is no continuous Fermi surface, only 'Fermi arcs'. (b) Photoemission from a FL* model in Ref. [61]. The Fermi surface consists of hole pockets of total area p, with intensity enhanced along the observed Fermi arcs, but suppressed on the 'back' sides of the pockets.

- The frequency and temperature dependence of the optical conductivity has a Fermi liquid form $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$ [1]. This Fermi liquid form is present although the overall prefactor corresponds to a carrier density p.
- Magnetoresistance measurements obey Kohler's rule [2] with $\rho_{xx} \sim \tau^{-1} \left(1 + aH^2T^2\right)$, again as expected by Fermi pocket of long-lived quasiparticles.
- Density wave modulations have long been observed in STM experiments [68] in the region marked DW in Fig. 1b. Following theoretical proposals [69, 70], a number of experiments [3–7] have identified the pattern of modulations as a d-form factor density wave. Computations of density wave instabilities of the FL* metal lead naturally to a d-form factor density wave, with a wavevector similar to that observed in experiments [71].
- Finally, very interesting recent measurements by Badoux *et al.* [8] of the Hall co-efficient at high fields and low T for $p \approx 0.16$ in YBCO clearly show the absence of DW order, unlike those at lower p. Furthermore unlike the DW region, the Hall co-efficient remains positive and corresponds to a density of p carriers. Only at higher $p \approx 0.19$ does the FL Hall co-efficient of $1 + p$ appear. A possible explanation is that the FL* phase is present in the doping regime $0.16 < p < 0.19$ without the appearance of DW order. In Fig. 1b, this corresponds to the T^* boundary extending past the DW region at low T.

6. Conclusion

We have described here the striking difference between the metallic states at low and high hole density, p, in the cuprate superconductors (see Fig. 1b). A theory for these states, and for the crossover between them, is clearly a needed precursor to any quantitative understanding of the high value of T_c for the onset of superconductivity.

At high p, there is strong evidence for a conventional Fermi liquid (FL) state. This is an 'unentangled' state, and its wavefunction is adiabatically connected to the free electron state which is a product of single-particle Bloch waves. The Fermi surface has long-lived fermionic excitations with charge $+e$ and spin $S = 1/2$. The volume enclosed by the Fermi surface is $1 + p$, and this obeys Luttinger's theorem. Such a Fermi surface is seen clearly in photoemission experiments [44], and also by the value of the Hall co-efficient [8].

At low p, in the PG regime, the experimental results pose many interesting puzzles. Numerous transport measurements [1, 2, 67], and also the remarkable recent Hall co-efficient measurements

of Badoux *et al.* [8] at low T and $p \approx 0.16$, are consistent with the presence of a Fermi liquid, but with a Fermi volume of p, which is not the Luttinger value. We described basic aspects of the theory of the fractionalized Fermi liquid (FL*) which realizes just such a Fermi surface. Long-range quantum entanglement, and emergent gauge fields, are necessary ingredients which allow the FL* metal to have a Fermi surface enclosing a non-Luttinger volume. The FL* metal also leads to a possible understanding [71] of the density wave (DW) order found at low temperatures in the pseudogap regime [3–7].

Assuming the presence of the distinct FL and FL* metals at high and low p, we are faced with the central open problem of connecting them at intermediate p. Although neither metal has a broken symmetry, the presence of emergent gauge fields in the FL* implies that there cannot be an adiabatic connection between the FL* and FL phases at zero temperature. So a quantum phase transition must be present, but it is not in the Landau-Ginzburg-Wilson symmetry-breaking class. We need a quantum critical theory with emergent gauge fields for the FL-FL* transition, and this can possibly provide an explanation for the intermediate 'strange metal' (SM) noted in Fig. 1b. Examples of FL-FL* critical theories have been proposed [45, 72], but a deeper understanding of such theories, and their connections to experimental observations in the SM, remain important challenges for future research.

Competing Interests. The author declares that he has no competing interests.

Funding. This research was supported by the NSF under Grant DMR-1360789. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation.

Acknowledgements. I thank Andrea Allais, Debanjan Chowdhury, Séamus Davis, Kazu Fujita, Antoine Georges, Mohammad Hamidian, Cyril Proust, Matthias Punk, and Louis Taillefer for numerous fruitful discussions on theories, experiments, and their connections.

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